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MATHEMATICAL ASTRONOMY.

The Collected Mathematical Works of George William Hill. Vol. i. Pp. xviii+363. (Washington: The Carnegie Institution, 1905.)

IT is a rare mind that can handle the cumbrous developments of practical astronomy and leave uppermost with a reader the impression of variety, ease, and polish; and curiosity will be felt as to the circumstances which have developed Hill's remarkable powers. From an interesting introduction to the present volume by M. Poincaré we learn that he spent three years at Rutgers College, New Jersey, under a certain Dr. Strong. Dr. Strong "était un homme de tradition, un laudator temporis acti; pour lui Euler était le Dieu des Mathématiques, et après lui la décadence avait commencé; il est vrai que c'est là un dieu que l'on peut adorer avec profit," and if it led Hill to the study of originals, we may overlook the depreciation of the moderns. From New Jersey he went to Cambridge to continue his studies at Harvard; very soon here, by a paper contributed for a prize to a mathematical miscellany, he attracted the notice of Runkle, the editor, who was Newcomb's predecessor at the office of the *American Ephemeris*. Hill became attached to the *Ephemeris* as computer, and remained in discharge of these duties for thirty-two years. At first he worked at his own home, as was then the custom; but under Newcomb's management, and in order to complete his theory and tables of Jupiter and Saturn he lived for some years at Washington, incessantly absorbed in his task. "The only defect of his make-up of which I have reason to complain," Newcomb has written, "is the lack of the teaching faculty." In 1892 he withdrew to the little farm where his boyhood was passed, and where he still lives, asking nothing but the liberty to continue his labours.

The present volume carries us up to 1881, and includes most, but by no means all, of his best known papers. The essay which attracted Runkle's notice is No. 3, "On the Conformation of the Earth," and was written at the age of twenty-three. It is perhaps not of any permanent importance, yet it is marked by the clearness and the firm hand of his later writings and the same salutary determination that theory should give an account of itself arithmetically. It is natural to compare it with Stokes's memoir "On the Variation of Gravity," written some twelve years before, when he also was a young man, and the comparison shows strikingly how Stokes is the physicist and Hill the analyst.

The two great memoirs by which Hill is best known are No. 29, "On the Part of the Motion of the Lunar Perigee which is a Function of the Mean Motions of the Sun and Moon," and No. 32, "Researches in the Lunar Theory." These writings have been greatly praised, but it seems impossible to praise them too highly, whether for their difficulties or the way these are overcome, or the greatness of the advance which their solution implies. The latter paper was the first which threw any real light upon the general problem of three bodies, and it is well worth notice how large a part arithmetic plays in its success. The analysis is pregnant in the extreme, but it is the actual calculation of a whole sequence of periodic orbits which a moon might occupy that gives it shape and name.

If this memoir may be said to be the first significant word on the problem of three bodies, the former one, on the motion of the lunar perigee, seems to be almost the last word on a question that had outrun calculation from Newton's day to Delaunay's. It is doubtful whether the more determined effort to calculate this quantity was made by Newton or by Delaunay, but though naturally the degrees of approximation they attained were very different, they had this in common, that they proved the inadequacy of the methods employed. Hill first, with the smoothness of a conjurer, gives form to the intractable equations, and then shows how the solution is contained in a certain transcendental equation, an infinite determinant. It affords striking evidence of Hill's power to contrast his treatment of this determinant with that of Adams, who followed a similar route, *sed longo intervallo*, as he said himself. The complexity arising from an infinite sequence of equations might seem to preclude any general conclusions from being drawn, but Hill uses this very feature in the most beautiful manner to derive the eliminant in a transcendental form in the shape $\cos \pi c =$ a known quantity, and from this equation determines c, the required ratio. The secret of the success is now apparent. c is nearly equal to unity; hence it is very much easier to approximate to $\cos \pi c$, where we are in the neighbourhood of a stationary value, than to c directly; and though the difficulty recurs when we seek to find the arc πc from a cosine in the neighbourhood of its minimum, it is then an insignificant one, for we are past the true complexities of the problem.[1]

The remaining papers are naturally not of equal moment with these, but we may be grateful to the Carnegie Institution for making them accessible in the present collection. Several of them arose in connection with Hill's duties as computer to the *Ephemeris*, but even on such hackneyed subjects as eclipse computing and reduction of star places he has something good to say. He is a true artist; *nullum quod tetigit non ornavit*. Of considerable general interest are No. 18, "Remarks on the Stability of Planetary Systems," and No. 14, "A Method of Computing Absolute Perturbations," which contains a rescension of Hansen. Even the smaller papers, like No. 22, "On the Solution of the Cubic and Biquadratic Equations," are usually marked by some analytical felicity that makes one wish that Hill had been able to bring his great powers to bear upon a material not so invariably intractable and overloaded with tradition, and limited in its problems, as practical astronomy. But if we feel that his hand

[1] The point of this approximation is put somewhat incorrectly by M. Poincaré in his introduction.

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is subdued to what it works in, we feel, too, that that is an essential ingredient of his success, and that with less complete absorption his work might have been less brilliant as well as less convincing.

R. A. S.

ULTRAMICROSCOPIC STUDIES OF THE COLLOIDS.

Zur Erkenntniss der Kolloide. Ueber irreversible Hydrosole und Ultramikroskopie. By Richard Zsigmondy. Pp. vi+185. (Jena: Gustav Fischer, 1905.) Price 4 marks.

THIS work forms a valuable addition to the literature of the colloids, giving as it does an authoritative account of the results obtained through the application of the method of ultramicroscopy to the study of solutions of colloids.

A brief account is first given of the nature and properties of colloid solutions or hydrosols. At the outset the author refers to the difficulty of giving a satisfactory definition of the term "solution." He adopts homogeneity as the most universal characteristic of solutions. The definition of homogeneity will naturally vary according to the delicacy of the methods employed to test it. By means of the method of ultramicroscopy devised by Zsigmondy and Siedentopf, the majority of colloid and even some crystalloid solutions can be shown to be optically heterogeneous. Every increase in the accuracy of the methods of examination would lead to a further limitation of the term "solution." In order to include the colloids Zsigmondy defines solutions as uniform distributions of solids in fluids, which are transparent to ordinary light, and not separable into their constituents by the action of gravity or by filtration.

In order to gain a clearer idea of the nature of colloid solutions, it is necessary to find criteria for distinguishing such solutions from those of crystalloids on the one hand and from suspensions on the other. Zsigmondy only refers very briefly to the distinguishing characteristics of the former, as this subject has been previously treated by Bredig in his monograph on "Inorganic Ferments." He deals more fully with the properties of colloid solutions which distinguish them from suspensions. In this connection he mentions the following as the chief features distinguishing colloid solutions from suspensions:—

(1) The particles in colloid solutions are much smaller than in suspensions. In colloid solutions the average diameter of the particles varies from 5 to 20 $\mu\mu$. This difference is, however, one only of degree.

(2) Many colloids are capable of undergoing irreversible changes. Separation of a metal from its colloid solution may be readily brought about by the withdrawal of water or the addition of electrolytes. In this process the metal has undergone an irreversible alteration or coagulation. For the reformation of the colloid solution, chemical or electrical means must be employed. In the case of suspensions, on the other hand, sedimentation rapidly takes place under the influence of gravity, and its rate is little influenced by the withdrawal of water or by the addition of electrolytes. The suspension may be reformed by purely mechanical means.

(3) Alterations in the total energy of the system are frequently associated with the process of coagulation. These have been measured in several cases by means of the calorimeter.

(4) Colloids in solutions are capable of undergoing reactions with one another, which closely simulate purely chemical reactions.

The next section of the book deals with the classification of colloid solutions or hydrosols. The classifications of the hydrosols have been based on two principles, namely, the size of the particles and the reversibility or irreversibility of the hydrosol (Hardy). On plate i. the author gives a graphic representation of a classification of colloids founded on these principles. The reversible colloids differ from the irreversible in not being readily coagulated by the addition of electrolytes. It is noteworthy that irreversible colloids may be partially protected from the coagulating action of electrolytes by the addition of a reversible colloid to their solutions. Great quantitative differences are found to exist in the extent of protection given by different reversible organic colloids to irreversible gold hydrosols.

A historical account of the preparation and properties of irreversible colloid solutions occupies the next section of the book.

The author next gives an interesting account of the development of the method of ultramicroscopy by Siedentopf and himself. A full description is also given of the necessary apparatus and of the method of using it.

The succeeding sections give details of the results of his own researches on gold hydrosols. By means of the ultramicroscope he was enabled to determine approximately the average size of the gold particles, their colour, and the rapidity of their movements both translatory and oscillatory. The limit of size determinable by the ultramicroscope appears to be about 6 $\mu\mu$ in the case of gold hydrosols. Still smaller particles (amicrones) are also present in gold hydrosols. Their presence may be proved by the coagulation of the hydrosols on the addition of electrolytes.

An excellent summary is also given of the results obtained by other observers through examination of various colloid solutions by means of the ultramicroscope.

Brief reference only is made to some points of great theoretical interest, namely, the causes of the stability of colloid solutions, and the mechanism of their formation.

The book concludes with a short summary of what is known with regard to the products of coagulation of colloid solutions.

The work as a whole is to be regarded as a valuable monograph indispensable for those interested in the ultramicroscope and its applications.

J. A. MILROY.

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